Evaluation of building foundation response due to tunnelling-induced settlements

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**ABSTRACT**

The construction of bored tunnels in soft ground inevitably causes ground movements which may damage utilities or affect adjacent buildings. Current risk assessment strategies involve either simplifying the building structure as an equivalent beam, or conducting sophisticated, and often time-consuming, numerical analyses simulating the superstructure and foundation response during the tunnelling process. This paper presents a novel and efficient approach which couples the superstructure stiffness with a piled foundation analysis tool, for the evaluation of holistic building response near tunnelling operations. The proposed approach considers the complete three-dimensional tunnelling-induced displacement field, including longitudinal displacements ahead of the tunnel heading and transverse displacements around the tunnel cross-section. This is illustrated and validated through comparisons with a hypothetical scenario and a case study in the Shenzhen Metro project. Since the approach is computationally less demanding than finite element or finite difference analyses, it can be extended to consider multiple structures along the tunnel alignment. The approach also promotes effective communication and division of labour between structural and geotechnical engineers in the integrated assessment of building response.

1 INTRODUCTION

As urban areas become more and more congested, there are increasing needs to construct new infrastructure underground, and in close proximity to existing structures in major cities around the world. In particular, as an efficient and environmentally friendly mode of transportation, construction of underground railway systems is a very popular means to meet the demands of public transportation. Construction of underground railway systems often involves tunnelling activities in densely developed urban areas. In Hong Kong, for example, the planning and construction of several new railway lines has led to new tunnels being constructed in densely-populated areas, adjacent to existing buildings and structures. These existing buildings are founded on a variety of foundation systems such as shallow footings, groups of frictional piles or end bearing piles. The impacts of tunnelling operations close to these foundation systems need to be evaluated to ensure safety and serviceability of the nearby structures.

Damage to buildings near tunnelling works can jeopardize safety of the existing structure, and lead to significant economic loss and project delay. From the risk management perspective, the method of building risk assessment usually follows a staged process: a preliminary assessment based only on the slope and settlement of the free-field ground surface; a second-stage assessment on the strains likely experienced by the building approximated as an equivalent beam; and the third-stage assessment through detailed numerical analysis of the building, if it is classified as being at intolerable risks under the conservative preliminary and second-stage assessments.

Study on piled building response to tunnelling-induced ground movements involves complex interactions among the superstructure, its foundation, and the soil around the tunnelling operations. The aforementioned second-stage assessment assumes that the building behaves as an equivalent beam, which is not representative of structures on piled foundations. On the other hand, the third-stage assessment can only be performed using sophisticated three-dimensional finite element or finite difference analyses, which are time-consuming and
require highly-skilled numerical analysts. This paper presents a newly-developed efficient analytical approach, which takes into consideration the three-dimensional displacement field around the tunnel and in front of the heading, the orientations and geometries of the building and foundations, soil-structure interaction effects and stiffness of the superstructure. Meanwhile, the approach is significantly more efficient and less computationally demanding than complete three-dimensional finite element analyses, and it is possible to extend the approach to assess the response of multiple structures along the tunnel alignment.

2 PROPOSED METHODOLOGY

Analysis of tunnelling effects on piled foundations can be performed with the ‘two-stage approach’ (Chen et al. 1999; Loganathan et al. 2001). According to this approach, the free-field (greenfield) soil movements around the tunnels are evaluated, ignoring the presence of piles in the first stage. The estimated displacement field is then imposed onto the piles in the second stage to evaluate their response. The current study extends this method to consider three-dimensional effects and the influence of superstructure stiffness on the foundation behaviour. The proposed approach consists of three components, namely evaluation of three-dimensional displacement field around the tunnel, modelling of pile group foundation, and incorporation of building stiffness in the foundation model.

2.1 Greenfield displacement induced by tunnelling

A common approach to evaluate tunnelling-induced displacements is the empirical method proposed by Peck (1969) and Schmidt (1969), which describes the transverse surface settlement trough above the tunnel using a Gaussian distribution curve. The method is popular due to its simplicity and the fact that it matches in-situ measurements reasonably well at many sites. Attewell et al. (1986) assumed, as later justified by Mair et al. (1993) and others, that the subsurface soil settlements also resemble a Gaussian curve. On the other hand, closed-form analytical solutions were developed by Sagaseta (1987) by considering soil displacements around the tunnel as a strain-controlled problem, while Rowe & Kack (1983) introduced the ‘gap’ parameter, taking into account the tunnelling methods and lining geometry. It should be noted that these analytical solutions were mainly developed based on concepts of cavity expansion (contraction) theories, and in most cases only plane-strain (two-dimensional) solutions have been established.

To evaluate the three-dimensional displacement field, Attewell et al. (1986) presented the solutions for subsurface displacements in the transverse direction, and also settlements along the longitudinal direction of the tunnel, by assuming the former as a Gaussian curve and that the latter follow a cumulative probability function. The surface settlement vertically above the tunnel face is assumed to be 50% of the maximum settlement, which is the long-term transverse settlement. They showed that without volume change, and when the deformation at the tunnel is approximated as a linearly-translating point source of ground loss, the displacements in three directions at any location can be evaluated by:

\[ w = \frac{V_s}{\sqrt{2\pi i}} \exp \left( -\frac{y^2}{2i^2} \right) \left[ G \left( \frac{x - x_f}{i} \right) - G \left( \frac{x - x_i}{i} \right) \right] \]  
\[ v = -\frac{n}{z_0 - z} yw \]  
\[ u = \frac{nV_s}{2\pi(z_0 - z)} \exp \left( -\frac{y^2}{2i^2} \right) \left[ \exp \left( \frac{-(x - x_i)^2}{2i^2} \right) - \exp \left( \frac{-(x - x_f)^2}{2i^2} \right) \right] \]

where \( u, v \) and \( w \) are the displacements in \( x, y \) and \( z \) directions, respectively, with \( x \) being the direction of tunnel advance; \( V_s \) is the volume of settlement trough per unit distance of tunnel advance; \( x_i \) is the starting position of the tunnel where \( x_f \) is the tunnel face position; \( z_0 \) is usually taken as the depth of tunnel axis, \( G(x) \) denotes the cumulative probability function, and \( n \) may be taken as 1 according to Attewell et al. (1986). \( i \) is the horizontal distance between tunnel centre-line and the point of inflexion of the settlement trough, and is usually taken as 0.5\( z_0 \). Making use of Equations (1) to (3), the three-dimensional greenfield displacement field around the tunnel can be defined. The induced displacements at the pile locations are of particular
importance, and this is denoted by the vector $\mathbf{u}^{gf}$ herein. For example, the induced displacements at location $i$ can be represented by $\mathbf{u}^{gf}_i = \{u_i, v_i, w_i\}$.

2.2 Consideration of superstructure stiffness in foundation analysis

One of the novel features of the proposed analysis tool is the direct coupling of superstructure stiffness into the pile group analysis through a ‘condensed’ structure matrix, which eliminates the need to approximate the superstructure as an equivalent beam or slab on top of the foundation (e.g. Potts & Addenbrooke 1997; Franzius et al. 2006). It capitalises on the fact that in many building projects, structural engineers construct building models using finite element packages as part of their design process. The complete structural model will include all the members in the building structure. Using these models, a condensed structure matrix, $\mathbf{K}^S$, can be generated by applying a unit displacement at each column in sequence, thus extracting the reaction forces at all of the other supports due to the unit displacement. For example, the component $K^S_{ij}$ in the condensed matrix represents the reaction force at support $i$ due to a unit displacement applied at support $j$ (Figure 1). Unlike the complete structural stiffness matrix, the condensed structure matrix is fully populated, with a size of $n \times n$, where $n$ is the number of columns or supports connecting the superstructure and the foundation, multiplied by the degrees of freedom at the supports. The condensed matrix can then be directly coupled with the foundation model to be discussed in the next section.

\[ (\mathbf{K}^P + \mathbf{K}^R) \mathbf{u} = \mathbf{p}^e + \mathbf{p}^g \]  

where $\mathbf{K}^P$ is the structural stiffness matrix of the piles modelled as beam elements, $\mathbf{u}$ is the vector of raft and pile displacements at the nodes, $\mathbf{p}^e$ is the external loading vector and $\mathbf{p}^g$ is the soil reaction to the pile and raft displacements. On the other hand, for the superstructure to be in equilibrium, the following can be written:
\[ \mathbf{K}^s \mathbf{u} = \mathbf{p}^{f_{dn}} + \mathbf{p}^w \]  

where \( \mathbf{K}^s \) is the condensed superstructure stiffness matrix described in Section 2.2, \( \mathbf{u} \) is the vector of column displacements, which is equal to the displacements at the corresponding foundation nodes connected to the columns, \( \mathbf{p}^{f_{dn}} \) is the interaction force of the foundation acting on the superstructure, and \( \mathbf{p}^w \) is the loading due to the self-weight and live loads acting on the structure. \( \mathbf{p}^w \) represents the gravity loads assuming no interaction with the foundation, which can be obtained from the support reactions assuming zero displacements at the supports. Since \( \mathbf{p}^s \) and \( \mathbf{p}^{f_{dn}} \) are action-reaction forces, they have equal magnitude but opposite signs. Therefore, rearranging and combining Equations (4) and (5) results in the governing equation of the coupled superstructure-foundation behaviour:

\[ (\mathbf{K}^p + \mathbf{K}^r + \mathbf{K}^s)\mathbf{u} = \mathbf{p}^w + \mathbf{p}^g \]  

When only elasticity is considered, the pile node displacement is also equal to the soil displacements at the same location (no slip). In this case, \( \mathbf{p}^g = -\lambda^{-1}\mathbf{u} \), where \( \lambda \) is the flexibility matrix evaluated using elastic solutions (e.g. Mindlin 1936). To simulate soil nonlinearity in the formulation, a slip element (plastic slider) can be incorporated into the continuum solution to limit the contact stresses between the soil and the pile shafts/bases, and between the raft and the soil underneath. In this case, \( \mathbf{u} \) consists of the continuum displacement and plastic interface displacements, and details of this formulation have been discussed in Leung (2010). The additional influence induced by nearby tunnelling operations can be incorporated into Equation (6) as an additional force represented by \( \lambda^{-1}\mathbf{u}^{gf} \), with \( \mathbf{u}^{gf} \) being the free-field tunnelling displacements described earlier. \( \lambda^{-1} \) may also be interpreted as the elastic soil stiffness matrix in the formulation.

The formulation presented above can be easily implemented in most programming languages (e.g. MATLAB, Mathematica), and once the foundation geometry is properly defined, the matrices can be assembled and evaluated in a few minutes in a typical desktop computer. Although compilation of \( \mathbf{K}^s \) requires the complete finite element model of the superstructure, this is usually available from the structural engineers as part of the structural design process, and minimal extra effort is required to extract the components of the condensed matrix. This process also promotes effective communication between geotechnical and structural professions, without the needs for geotechnical engineers to construct the structural model, or the structural engineers to make crude approximations of the soil behaviour. Moreover, the calculation time is greatly reduced compared to full three-dimensional finite element modelling of the entire superstructure, foundation piles, and the tunnelling process in the soil domain.

3 VERIFICATION CASE STUDIES

Two cases are presented herein for the verification of the proposed method. The first case is a hypothetical scenario reported by Loganathan et al. (2001) and Kitiyodom et al. (2005), where the tunnelling-induced displacements in transverse direction is considered, and superstructure is not included. The focus of the first case is to verify the proposed approach under simple conditions and to compare its results with analyses by previous researchers. In the second case, the proposed approach is compared with three-dimensional finite element modelling performed by Zhu et al. (2013) for a section of the Shenzhen Metro extension, where tunnelling operations were conducted next to a 7-storey building. The influence of building stiffness on the foundation behaviour is also illustrated in this case.

3.1 Two-dimensional hypothetical scenario

Figure 2 shows the plane-strain scenario described by Loganathan et al. (2001) and analysed by Kitiyodom et al. (2005), where a 20-m deep, 6-m diameter tunnel was excavated 4.5 m away from a group of 4 piles. The geometry of the pile group, elastic material properties of the soil and pile materials are also shown in Figure 2.

The hypothetical scenario only involved displacements in the transverse direction, with no superstructure connected with the foundation. Therefore, it was analysed using the proposed analysis method with a two-dimensional displacement field and \( \mathbf{K}^s \) was not evaluated for comparison purposes. Figure 2 shows that the analysis results using the proposed method compare favourably with those by Kitiyodom et al. (2005) regarding the estimated lateral pile deflections, and induced pile bending moments and axial forces.
3.2 Three-dimensional analysis of building near Shenzhen Metro extension project

Zhu et al. (2013) presented the three-dimensional finite element analyses of a 7-storey building near the Xixiang Station, along Luobao line of the Shenzhen Metro extension project (Figure 3(a)). The building is a reinforced concrete (C25) structure with storey height of 3 m, floor dimensions of 20 m × 14 m, and column dimensions of 500 mm × 500 mm. The beams are 200 mm (width) × 300 mm (depth), while the slab thickness is 100 mm. These structural details are modelled in the current study using SAP2000, to obtain the $K_s$ matrix through the procedure described in Section 2.2.

Each column is supported by a 14.7-m concrete long pile, with dimensions of 500 mm × 500 mm. Zhu et al. (2013) did not report or model any details of a pile cap, and for comparison purposes, this is also not simulated in the current study. About 13 m to the edge of the existing building, a 6-m diameter tunnel was being excavated by tunnelling boring machine at a depth of 19 m.

The soil profile at the site consists of 6 m of fill, 20 m of clay and silty fine sand overlying completely to moderately weathered mudstone, and the rockhead is 40 m below the ground surface. The Young’s modulus is shown in Figure 3(d), and $c'$ ranges from 21-28 kPa and $\phi'$ from 31-33° for most soil layers. In the current study, the pile interaction effects are simulated through the flexibility matrix ($\lambda$) using the Mindlin’s solution, which was developed for homogeneous half space. To simulate the effects of non-homogeneous soils, the average soil modulus representative of the two interacting pile elements is adopted when evaluating $\lambda$ (Poulos 1979). Meanwhile, influence of the rockhead is simulated by the Steinbrenner approximation, where $\lambda_{i,j}$ is modified to simulate presence of an underlying stiff stratum (Poulos & Davis 1968).
Field monitoring data of ground settlements shows that the volume loss is approximately 2% (Figure 4(a)) due to the tunnelling operations, and this value is adopted in Equations (1) to (3) to estimate the displacement field around the tunnel. Zhu et al. (2013) did not report measurements of building displacements. Instead, they presented the estimated amount of structure twisting ($T_w$) using the finite element model, which is defined as:

$$T_w = \frac{(S_{A1} - S_{D1})/B - (S_{A5} - S_{D5})/B}{L}$$

(7)

where $S_{A1}, S_{D1}, S_{A5}, S_{D5}$ are the tunnelling-induced settlements at A1, D1, A5 and D5, respectively. $B$ and $L$ are the width and length of the building footprint (Figure 4). As the tunnel advances, approaches and passes the building (from the side), the amounts of differential settlements, and hence twisting, will change accordingly. $T_w$ estimated by Zhu et al. (2013) at various stages of tunnel excavation, and the corresponding estimates using the approach proposed in the current study, are shown in comparison in Figure 4(b). The differences between the two approaches are generally less than 30%, and it should be noted that the proposed approach is computationally less demanding and the analyses completed in a few minutes. The obtained foundation settlements can be easily transferred back to the SAP2000 model, in order to assess the superstructure performance due to these foundation movements. Figure 4(b) also shows the estimated twisting when the superstructure stiffness ($K^s$) is not considered in the analyses, in which case the maximum $T_w$ is overestimated by about 3.5 times. This underlines the importance of including superstructure effects when assessing the foundation response.

The stiffness of superstructure also has important implications on the forces experienced by the piles due to tunnelling. Figure 5(a) and (b) shows the tunnelling-induced settlements and axial forces and bending moments evaluated for piles D4 and D5 with and without considering $K^s$ in the analyses, respectively. It
should be noted that the effects due to self-weights of the structure are removed in these plots in order to highlight the effects of the nearby tunnel excavation. If the superstructure is not considered, the settlement at D4 is minimal as it is located 18 m away (horizontally) from the tunnel centre-line. However, considering the effects $K_s^2$, the stiff superstructure causes the pile group to tilt like a rigid body, thereby increasing the settlements and also axial forces at D4 and reducing those at D5. In fact, the tunnel excavation induces a tensile force on D5, although it is still in compression when the structure weights are also included. Meanwhile, the displacements and forces experienced at the pile heads can be transmitted to structural engineers and input as boundary conditions at column supports of the superstructure model, in order to evaluate response of all other structural elements.

![Figure 4: (a) Ground settlement measurements and comparison with Gaussian curve; (b) Building twisting estimated by 3D FEM (Zhu et al. 2013) and the proposed approach in this study](image)

![Figure 5: Tunnel-induced settlements, axial forces and bending moments at D4 and D5: (a) with $K_s^2$ and (b) without $K_s^2](image)
4 CONCLUSIONS

This paper presents an efficient analysis approach to couple superstructure stiffness to the modelling of piled foundations under the effects of nearby tunnelling operations. The approach promotes effective communication and information exchange between structural and geotechnical engineers, both of which can incorporate their expertise in the holistic assessment. The proposed approach compares favourably with other simulation methods, such as three-dimensional finite element simulation of the entire structure and soil domain, which is much more demanding in computational efforts. Considering the efficiency of the approach, it can be easily extended to consider multiple building structures along the tunnel alignment, which is a typical scenario for tunnelling projects in urban settings, greatly enhancing its value in conducting regional geotechnical and structural risk assessments in tunnelling alignment designs.

ACKNOWLEDGEMENTS

The work presented in this paper is an extension of a Final Year Project at The Hong Kong Polytechnic University. The authors would like to acknowledge the constructive comments by Ir C.K. Chan of Atkins as the project moderator.

REFERENCES